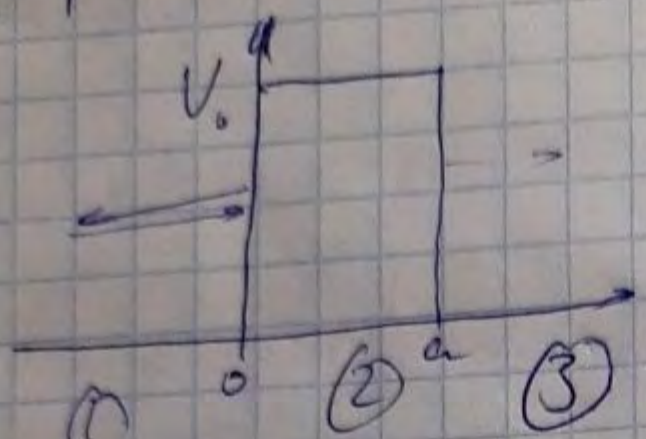
b/3 1



1: $E < V_0$:

$$\frac{d^2\psi_1}{dx^2} + \underbrace{\frac{2m}{\hbar^2}E}_{=k^2}\psi_1(x) = 0$$

$$\lambda^2 + k^2 = 0$$

$$\lambda = \pm\sqrt{-k^2} = \pm ik$$

$$\psi_1 = \underset{\text{III}}{C_1 e^{ikx}} + C_2 e^{-ikx}$$

$$\psi_1 \doteq \underbrace{A e^{ikx}}_{\longrightarrow} + \underbrace{B e^{-ikx}}_{\longleftarrow}$$

$$R = \frac{|B|^2}{|A|^2}$$

2: $$\frac{d^2\psi_2}{dx^2} + \underbrace{\frac{2m}{\hbar^2}\underbrace{(E - V_0)}_{<0}}_{= -\varkappa^2}\psi_2 = 0$$

$$\frac{d^2\psi_2}{dx^2} - \varkappa^2\psi_2 = 0$$

$$\lambda^2 = \varkappa^2 \Rightarrow \lambda = \pm\varkappa$$

$$\psi_2^{I} = Ce^{\varkappa x} + De^{-\varkappa x}$$
$$\psi_2^{II} = Ee^{\varkappa x} + Fe^{-\varkappa x} \quad \Big| \Rightarrow$$

$$C + D = \frac{i}{k}\varkappa C + \frac{i}{k}\varkappa D$$

$$ikCe^{\varkappa a - ika} + ikDe^{-\varkappa a - ika} = \varkappa Ce^{\varkappa a - ika} - \varkappa De^{-\varkappa a - ika}$$

3: $\psi_3 = Ge^{ikx} + 0$

$A = 1$

$1 + B = C + D$

$ik - ikB = \varkappa C - \varkappa D$

$B = C + D - 1$

$B = 1 - \frac{\varkappa C - \varkappa D}{ik}$

$Ge^{ika} = Ce^{\varkappa a} + De^{-\varkappa a}$

$ikGe^{ika} = \varkappa Ce^{\varkappa a} - \varkappa De^{-\varkappa a}$

$G = Ce^{\varkappa a - ika} + De^{-\varkappa a - ika}$

$G = \frac{\varkappa Ce^{\varkappa a - ika} - \varkappa De^{-\varkappa a - ika}}{ik}$

Скінченність: $C = 0$, $E = 0$

Неперервність в 0: $\psi_1|_{x=0} = {}^{I}\psi_2|_{x=0} \Rightarrow 1 + B = D$

$\psi_1'|_{x=0} = {}^{I}\psi_2'|_{x=0} \Rightarrow ik - ikB = -\varkappa D$

$ik - ik\beta = -\varkappa - \varkappa\beta \Rightarrow (-ik + \varkappa)\beta = -\varkappa - ik \Rightarrow$

$\Rightarrow \beta = \frac{ik + \varkappa}{ik - \varkappa}$

Неперервність в а: $\psi_2|_{x=a} = \psi_3|_{x=a} \Rightarrow$

$\Rightarrow Fe^{-\varkappa a} = Ge^{ika} \Rightarrow F = Ge^{ika + \varkappa a}$

$\psi_2'|_{x=a} = \psi_3'|_{x=a} \Rightarrow -\varkappa F e^{-\varkappa a} = ikGe^{ika} \Rightarrow$

$\Rightarrow G$ і $F \neq 0$

$R = \frac{|\underline{\psi_2}|^2}{|\underline{\psi_1}|^2} = \frac{|B|^2}{|A|^2} = 1; \quad T = \frac{|\psi_3|^2}{|\underline{\psi_1}|^2} = \frac{0}{1} = 0$

Тепер розглянемо $E > U_0$

1: $\psi_1 = Ae^{ikx} + Be^{-ikx}$

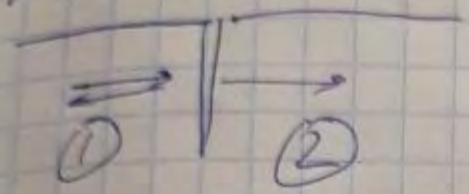
2: $\psi_2^I = Ce^{ixx} + De^{-ixx}$

$\psi_2^{II} = Ee^{ixx} + Fe^{-ixx}$

Неперервність в 0: $1 + B = C$

$ik - ikB = ixC$

№2



$V(x) = -V_0 \delta(x)$

① $\psi_1 = Ae^{ikx} + Be^{-ikx}$

② $\frac{d^2\psi_2}{dx^2} + \frac{2m}{\hbar^2}(\underbrace{E - V_0}_{>0})\psi_2 = 0$

$\psi_2 = Ce^{ikx}$; $\psi_1|_{t=0} = \psi_2|_{t=0} \Rightarrow 1 + B = C$

$\psi_2'|_{t=0} - \psi_1'|_{t=0} = ik(C + B - 1) = -\frac{2mV_0 C}{\hbar^2}$

$ik + 2B = -\frac{2mV_0}{\hbar^2}(1 + B)$

$ik + 2B = -\frac{2mV_0}{\hbar^2} - \frac{2mV_0}{\hbar^2}B$

$B\left(\frac{2\hbar^2 - 2mV_0}{\hbar^2}\right) = \frac{2mV_0 - ik\hbar^2}{\hbar^2} \Rightarrow$

$\Rightarrow B = \frac{2mV_0 - ik\hbar^2}{2\hbar^2 - 2mV_0}$; $C = \frac{2\hbar^2 + ik\hbar^2}{2\hbar^2 - 2mV_0}$

$R = \frac{|B|^2}{|A|^2}$, $|B| = \frac{\sqrt{4m^2V_0^2 + k^2\hbar^4}}{2\hbar^2 - 2mV_0} \Rightarrow R = \frac{4m^2V_0^2 + k^2\hbar^4}{(2\hbar^2 - 2mV_0)^2}$

$T = \frac{|C|^2}{|A|^2}$, $|C| = \frac{\sqrt{4\hbar^4 + k^2\hbar^4}}{2\hbar^2 - 2mV_0} \Rightarrow T = \frac{4\hbar^4 + k^2\hbar^4}{(2\hbar^2 - 2mV_0)^2}$

б/з 2

$$\hat{L} = [\hat{\vec{r}} \times \hat{\vec{p}}]$$

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}$$

$$\hat{L}_x = y\hat{p}_z - z\hat{p}_y$$

$$[\hat{L}_x, V(r)] = \hat{L}_x(V\varphi) - V\hat{L}_x(\varphi) =$$

$$= -iy\hbar \frac{\partial}{\partial z}(V\varphi) + iz\hbar \frac{\partial}{\partial y}(V\varphi) -$$

$$-V\left(-iy\hbar \frac{\partial \varphi}{\partial z} + iz\hbar \frac{\partial \varphi}{\partial y}\right) = iy\hbar V \frac{\partial}{\partial z} - i\hbar z \frac{\partial}{\partial y}$$

$$[\hat{L}_x, \quad] = \hat{L}_x\hat{L} - \hat{L}\hat{L}_x$$

$$[\hat{L}_x, V(r)]$$

$$=$$

$$=$$

$$=$$

$$[\hat{L}_x, \hat{L}^2] = [\hat{L}_x, \sum_e \hat{L}_e^2] = \sum_e [\hat{L}_x, \hat{L}_e \hat{L}_e] =$$

$$= \sum_e \{[\hat{L}_x, \hat{L}_e]\hat{L}_e + \hat{L}_e[\hat{L}_x, \hat{L}_e]\} =$$

$$= \text{[illegible]} = i\hbar \sum_{klm} \varepsilon_{klm}(\hat{L}_m \hat{L}_e + \hat{L}_e \hat{L}_m) =$$

$$= i\hbar \sum_{em} \varepsilon_{kem} \hat{L}_m \hat{L}_e + i\hbar \sum_{lm} \varepsilon_{kem} \hat{L}_e \hat{L}_m \Rightarrow$$

$$\Rightarrow [\hat{L}_k, \hat{L}^2] = i\hbar \sum_{em} (\varepsilon_{kem} + \varepsilon_{kme}) \hat{L}_m \hat{L}_e =$$

$$= i\hbar \sum_{em} (\varepsilon_{kem} - \varepsilon_{kem}) \hat{L}_m \hat{L}_e = 0$$

$$-i\hbar z \frac{\partial}{\partial y}$$

b/3 3

№1 $L^{-1} = \frac{1}{1+\frac{d}{dx}}$

$$\frac{1}{1+x} = 1 - x + x^2 + \dots + (-1)^n x^n$$

$$L^{-1} = \sum_{n=0}^{\infty} (-1)^n \frac{d^n}{dx^n}$$

$$L^{-1}\psi = \sum_{n=0}^{\infty} (-1)^n \frac{d^n\psi}{dx^n} = L\psi$$

$$\psi = e^{\lambda x}$$

$$L\psi = e^{\lambda x}(1 - \lambda + \lambda^2 + \dots (-1)^n \lambda^n) = L e^{\lambda x} \Rightarrow$$

$$\Rightarrow L = \frac{1}{1+\lambda}$$

3 умови скінченності

$\lambda = \pm im$, [redacted] $m \in R$

$$L = \frac{1}{1 \pm im} \cdot \frac{(1 \mp im)}{(1 \mp im)} = \frac{(1 \mp im)}{(1+m^2)}$$ - Власні значення

$\psi = c e^{\lambda x} = c e^{\pm imx}$ - Власні ф-ї

tside . . .

$\hat{L} = \frac{1}{1+\frac{d}{d\varphi}}$

$\psi = Ce^{\lambda\varphi} = Ce^{(\varphi+2\pi)\lambda} \Rightarrow e^{2\pi\lambda} = 1$

$\downarrow\downarrow$

$L = \frac{1}{1+\lambda}$, $\lambda = \pm im$, $m \in \mathbb{Z}$ - власні значення

Нормуємо

$\int_0^{2\pi} |\psi|^2 d\varphi = 1$; $|C|^2 \int_0^{2\pi} |e^{2m\varphi}| d\varphi = 1 \Rightarrow$

$\Rightarrow |C|^2 \cdot 2\pi = 1 \Rightarrow |C| = \sqrt{\frac{1}{2\pi}}$

$C = \sqrt{\frac{1}{2\pi}} e^{i\beta}$

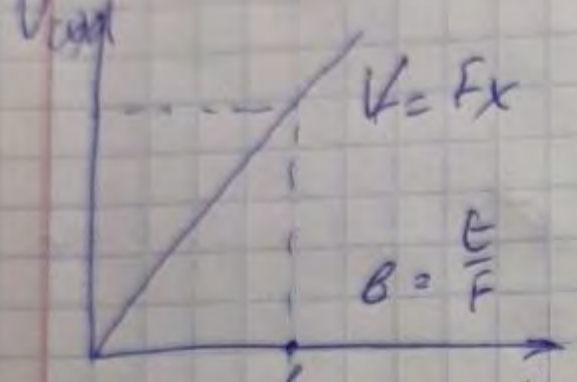
$\psi = \sqrt{\frac{1}{2\pi}} e^{i\beta} e^{\mp im\varphi}$ - власні ф-ї

utside

Д/З 4

$\varphi = Ax e^{-\lambda x}$

Нормуємо:



$$|A|^2 \int_0^b x^2 e^{-2\lambda x} dx = 1 \Rightarrow$$

$$|A|^2 \left[\left(x^2 e^{-2\lambda x} \cdot \frac{1}{2\lambda} \right)\Big|_0^b + \int_0^b \frac{x}{\lambda} e^{-2\lambda x} dx \right] =$$

$$= |A|^2 \left[-e^{-2b\lambda} \cdot \frac{b^2}{2\lambda} + \int_0^b \frac{x}{\lambda} e^{-2\lambda x} dx \right] =$$

$$= |A|^2 \left[-e^{-2b\lambda} \cdot \frac{b^2}{2\lambda} + \left(\frac{x}{2\lambda^2} e^{-2\lambda x} \right)\Big|_0^b + \int_0^b \frac{1}{2\lambda^2} e^{-2\lambda x} dx \right] =$$

$$= |A|^2 \left[-e^{-2\lambda b} \frac{b^2}{2\lambda} - \frac{b}{2\lambda^2} e^{-2\lambda b} + \frac{1}{4\lambda^3} (1 - e^{-2\lambda b}) \right] = 1$$

$$J_0 = \int_0^b A x e^{-2x} \left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + Fx \right] A x e^{-\lambda x} dx =$$

$$= -A^2 \frac{\hbar^2}{2m} \int_0^b (2x - 4\lambda x^2 + \lambda^2 x^3) e^{-2\lambda x} dx +$$

$$+ A^2 F \int_0^b x^3 e^{-2\lambda x} dx.$$

Розв'яжемо окремо кожен інтеграл:

$$2 \int_0^b x e^{-2\lambda x} dx = 2x e^{-2\lambda x} \cdot \frac{1}{-2\lambda} \Big|_0^b - \frac{1}{\lambda} \int_0^b e^{-2\lambda x} dx =$$

$$= -\frac{b}{\lambda} e^{-2\lambda b} - \frac{1}{2\lambda^2} (e^{-2\lambda b} - 1) = \frac{-E}{F\lambda} e^{-2\lambda \frac{E}{F}} - \frac{e^{-2\lambda \frac{E}{F}}}{2\lambda^2} + \frac{1}{2\lambda^2}$$

(16.32)

$$4\lambda\int_0^b x^2 e^{-2\lambda x}dx = 4\lambda x^2\frac{e^{-2\lambda x}}{2\lambda}\Big|_0^b - \frac{4}{\lambda}\int_0^b x e^{-2\lambda x}dx =$$

$$= -2b^2 e^{-2\lambda b} + 2\lambda\left(-\frac{b}{\lambda}e^{-2\lambda b} + \frac{1}{2\lambda^2} - \frac{e^{-2\lambda b}}{2\lambda^2}\right) =$$

$$= -2b^2 e^{-2\lambda b} - \frac{b}{2}e^{-2\lambda b} - \frac{e^{-2\lambda b}}{\lambda} + \frac{1}{\lambda}$$

$$\int_0^b x^3 e^{-2\lambda x}dx = -x^3\frac{e^{-2\lambda x}}{2\lambda}\Big|_0^b + \frac{3}{2\lambda}\int_0^b x^2 e^{-2\lambda x}dx =$$

$$= -b^3\frac{e^{-2\lambda b}}{2\lambda} + \frac{3}{8\lambda^2}\left(-2b^2 e^{-2\lambda b} - \frac{b}{2}e^{-2\lambda b} - \frac{e^{-2\lambda b}}{\lambda} + \frac{1}{\lambda}\right) =$$

$$= -b^3\frac{e^{-2\lambda b}}{2\lambda} - \frac{3}{4\lambda^2}b^2 e^{-2\lambda b} - \frac{3b}{16\lambda^2}e^{-2\lambda b} - \frac{3}{8\lambda^3}e^{-2\lambda b} + \frac{3}{8\lambda^3}$$

Теперь:

$$J_0 = -A^2\frac{\hbar^2}{2m}\left(-\frac{b}{\lambda}e^{-2\lambda b} - \frac{e^{-2\lambda b}}{2\lambda^2} + \frac{1}{2\lambda^2} + 2b^2 e^{-2\lambda b} + \frac{b}{2}e^{-2\lambda b} + \right.$$

$$+ \frac{e^{-2\lambda b}}{\lambda} - \frac{1}{\lambda} - b^3\lambda\frac{e^{-2\lambda b}}{2} - \frac{3}{4}b^2 e^{-2\lambda b} - \frac{3b}{16}e^{-2\lambda b} - \frac{3}{8\lambda}e^{-2\lambda b} + \left.\frac{3}{8\lambda}\right) +$$

$$+ A^2 F\left(\frac{3}{8\lambda^3} - b^3\frac{e^{-2\lambda b}}{2\lambda} - \frac{3}{4\lambda^2}b^2 e^{-2\lambda b} - \frac{3b}{16\lambda^2}e^{-2\lambda b} - \frac{3}{8\lambda^3}e^{-2\lambda b}\right) =$$

$$= -A^2\frac{\hbar^2}{2m}\left(\frac{e^{-2\lambda b}}{\lambda}\left(-b + 1 - \frac{3}{8}\right) + \frac{e^{-2\lambda b}}{\lambda^2}\left(-\frac{1}{2}\right) + \lambda e^{-2\lambda b}\left(\frac{b^3}{2}\right) + \right.$$

$$+ e^{-2\lambda b}\left(2b^2 + \frac{b}{2} - \frac{3}{4}b^2 - \frac{3b}{16}\right) + \frac{1}{2\lambda^2} + \frac{1}{\lambda}\left(-1 + \frac{3}{8}\right) +$$

$$\left. + A^2 F\left(\frac{3}{8\lambda^3} - b^3\frac{e^{-2\lambda b}}{2\lambda} - \frac{3}{8\lambda^3}e^{-2\lambda b} + \frac{e^{2b\lambda}}{\lambda^2}\left(-\frac{3}{4}b^2 - \frac{3b}{16}\right)\right)\right)$$

(16.32)

$$\frac{dJ_0}{dL} = -A^2\frac{\hbar^2}{2m}\Big[\Big(\frac{5}{8}-6\Big)\Big(-2b\frac{e^{-2Lb}}{L} - \frac{e^{-2Lb}}{L^2}\Big) - \frac{1}{2}\Big(-2b\frac{e^{-2Lb}}{L^2} - 2\frac{e^{-2Lb}}{L^3}\Big) + \frac{b^3}{2}\big(e^{-2Lb} - 2Lbe^{-2Lb}\big) - 2be^{-2Lb}\Big(\frac{5}{4}b^2 - \frac{5}{16}b\Big) - \frac{1}{L^3} + \frac{5}{8L^2}\Big] + A^2F\Big[-\frac{9}{8L^4} - b^3\Big(-2b\frac{e^{-2Lb}}{L} - \frac{e^{-2Lb}}{L^2}\Big) - \frac{3}{8}\Big(-2e\frac{e^{-2Lb}}{L^3} - \frac{3e^{-2Lb}}{L^4}\Big) + \Big(-\frac{3b}{16} - \frac{3}{4}b^2\Big)\Big(-2b\frac{e^{-2Lb}}{L^2} - 2\frac{e^{-2Lb}}{L^3}\Big)\Big]$$

$$- L^2\frac{\hbar^2}{2m}\Big[\Big(\frac{5}{8}-6\Big)\big(-2bL^2e^{-2Lb} - L^2e^{-2Lb}\big) - \frac{1}{2}\big(-2bL^2e^{-2Lb} - 2Le^{-2Lb}\big) + \frac{b^3}{2}\big(L^4e^{-2Lb} - 2L^5be^{-2Lb}\big) - 2bL^4e^{-2Lb}\Big(\frac{5}{4}b^2 - \frac{5}{16}b - L + \frac{5}{8}L^2\Big)\Big] + A^2F\Big[-\frac{9}{8} - b^3\big(-2bL^3e^{-2Lb} - L^2e^{-2Lb}\big) - \frac{3}{8}\big(-2bLe^{-2Lb} - 3e^{-2Lb}\big) + \Big(-\frac{3b}{16} - \frac{3}{4}b^2\Big)\big(-2bL^2e^{-2Lb} - 2Le^{-2Lb}\big) = 0$$

$$L^5\Big(\frac{A^2\hbar^2b^4e^{-2Lb}}{2m}\Big) + L^4\Big[-A^2\frac{\hbar^2}{2m}\Big(\frac{b^3}{2} - \frac{5}{8}b^3 + \frac{5}{32}b^2\Big)e^{-2Lb}\Big] + L^3\Big[-A^2\frac{\hbar^2}{2m}\Big(-\frac{5}{4}b^3 + 2b^2\Big)e^{-2Lb} + A^2F2b^4e^{-2Lb}\Big] + [\text{illegible}] + L^2\Big[-A^2\frac{\hbar^2}{2m}\cdot 2be^{-2Lb} + A^2F\Big(b^3 - \frac{3}{8}b^2 - \frac{3}{2}b^3\Big) + L\Big[-A\frac{\hbar^2}{2m}\big(e^{-2Lb} - 1\big) + A^2F\Big(\frac{3}{4}b + \frac{3b^2}{8}L + \frac{3}{2}b^2\Big)e^{-2Lb}\Big] + A^2F\Big(-\frac{9}{8}\Big) + \frac{9}{8}e^{-2Lb} = 0$$

Old outside.

Б/З 6

№1 $V(x) = \frac{m\omega^2 x^2}{2}$

$\psi_0 = Ae^{-\lambda|x|}$, $\psi_1 = Bxe^{-\beta|x|}$

0: Нормируем ф-ю ψ:

$$|A|^2 \int_{-\infty}^{+\infty} e^{-2\lambda|x|}\,dx = 1 \Rightarrow |A|^2 \left\{ \int_{-\infty}^{0} e^{2\lambda x}\,dx + \int_{0}^{+\infty} e^{-2\lambda x}\,dx \right) =$$

$$= |A|^2 \left(\frac{1}{2\lambda} e^{2\lambda x}\Big|_{-\infty}^{0} - \frac{1}{2\lambda} e^{-2\lambda x}\Big|_{0}^{\infty} \right) = |A|^2 \left(\frac{1}{2\lambda} + \frac{1}{2\lambda} \right) = 1 \Rightarrow$$

$$\Rightarrow A = \sqrt{\lambda}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{d^2}{dx} + \frac{m\omega^2 x^2}{2}$$

$$\hat{H}\psi_0 = -\frac{\hbar^2}{2m}\frac{d}{dx}\left[\frac{d}{dx}\left(Ae^{-\lambda|x|}\right)\right] + \frac{m\omega^2 x^2}{2} Ae^{-\lambda|x|} =$$

$$= -\frac{\hbar^2}{2m} A \frac{d}{dx}\left(-\lambda e^{-\lambda|x|}\frac{x}{|x|}\right) + \frac{m\omega^2 x^2}{2} Ae^{-\lambda|x|} =$$

$$= A\frac{\hbar^2}{2m}\lambda\left(-\lambda e^{-\lambda|x|} + e^{-\lambda|x|}\frac{|x| - \frac{x^2}{x}}{x^2}\right) + \frac{m\omega^2 x^2}{2} Ae^{-\lambda|x|} =$$

$$= -\frac{\lambda^2\hbar^2}{2m} Ae^{-\lambda|x|} + \frac{m\omega^2 x^2}{2} Ae^{-\lambda|x|}$$

$$J_0 = \int A e^{-\lambda|x|}\left[A e^{-\lambda|x|}\cdot\left(-\frac{\lambda^2\hbar^2}{2m}\right) + \frac{m\omega^2 x^2}{2} A e^{-\lambda|x|}\right]dx =$$

$$= \int A^2 e^{-2\lambda|x|}\left(-\frac{\lambda^2\hbar^2}{2m}\right)dx + \int A^2 e^{-2\lambda|x|}\cdot x^2 \cdot \frac{m\omega^2}{2}\,dx$$

Розвіяємо окремо:

$$\int_{-\infty}^{+\infty} x^2 e^{-2\lambda|x|}dx = \int_{-\infty}^{0} x^2 e^{2\lambda x}dx + \int_{0}^{\infty} x^2 e^{-2\lambda x}dx =$$

$$= \cancel{\frac{x^2 e^{2\lambda x}}{2\lambda}\Big|_{-\infty}^{0}} - \int_{-\infty}^{0}\frac{e^{2\lambda x}}{2\lambda}\,2x\,dx - \cancel{\frac{1}{2\lambda}x^2 e^{-2\lambda x}\Big|_{0}^{\infty}} +$$

$$+ \int_{0}^{\infty}\frac{x}{\lambda}e^{-2\lambda x}dx = -\frac{1}{\lambda}\int_{-\infty}^{0} x e^{2\lambda x}dx + \frac{1}{\lambda}\int_{0}^{\infty} x e^{-2\lambda x}dx =$$

$$= -\cancel{\frac{1}{\lambda}\left[\left(\frac{1}{2\lambda}x e^{2\lambda x}\right)\Big|_{-\infty}^{0}\right.} - \int_{-\infty}^{0}\frac{1}{2\lambda}e^{2\lambda x}dx\Big] +$$

$$+ \cancel{\frac{1}{\lambda}\left[\left(-\frac{1}{2\lambda}x e^{-2\lambda x}\right)\Big|_{0}^{\infty}\right.} + \int_{0}^{\infty}\frac{1}{2\lambda}e^{-2\lambda x}dx\Big] =$$

$$= \frac{1}{2\lambda^2}\int_{0}^{\infty} e^{-2\lambda x} - \frac{1}{2\lambda^2}\int_{-\infty}^{0} e^{2\lambda x}dx = \frac{1}{2\lambda^2}\left(\frac{1}{2\lambda}e^{+2\lambda x}\Big|_{-\infty}^{0} - \frac{1}{2\lambda}e^{-2\lambda x}\Big|_{0}^{\infty}\right) =$$

$$= \frac{1}{2\lambda^2}\cdot\frac{1}{\lambda} = \frac{1}{2\lambda^3}$$

$$\int_{-\infty}^{+\infty} e^{-2\lambda|x|}dx = \int_{-\infty}^{0} e^{2\lambda x}dx + \int_{0}^{\infty} e^{-2\lambda x}dx = \frac{1}{2\lambda}e^{2\lambda x}\Big|_{-\infty}^{0} -$$

$$- \frac{1}{2\lambda}e^{-2\lambda x}\Big|_{0}^{\infty} = \frac{1}{2\lambda} + \frac{1}{2\lambda} = \frac{1}{\lambda}$$

$\sqrt{2\alpha_1^3/\pi}$ (16.31)

...риводить до результату

$\ldots + \frac{\mu\omega^2}{\ldots}\Big)$. (16.32)

$$J_0 = -A^2 \frac{\lambda \hbar^2}{2m} + \frac{m\omega^2}{2} A^2 \cdot \frac{1}{2\lambda^3} = \left| A^2 = \lambda \right| =$$

$$= \frac{-\lambda^2 \hbar^2}{2m} + \frac{m\omega^2}{4\lambda^2}$$

$$\frac{dJ_0}{d\lambda} = 0 \Rightarrow -\lambda \frac{\hbar^2}{m} - \frac{m\omega^2}{2\lambda^3} = 0 \Rightarrow$$

$$\Rightarrow \frac{\lambda^4 \hbar^2}{m} + \frac{m\omega^2}{2} = 0 \Rightarrow \lambda = \left(-\frac{m^2\omega^2}{2\hbar^2} \right)^{\frac{1}{4}} =$$

$$= \sqrt{\frac{m\omega}{\hbar}} \cdot \sqrt[4]{-\frac{1}{2}}$$

$$E_0 = \frac{m\omega}{\hbar} \sqrt{-\frac{1}{2}} \cdot \frac{\hbar^2}{2m} - \frac{m\omega^2}{4m\omega} \sqrt{-2}\, \hbar \Rightarrow$$

$$\Rightarrow E_0 = \frac{\hbar\omega}{\sqrt{-2}} - \frac{\sqrt{-2}}{4} \hbar\omega = \frac{\sqrt{-2}}{4} \hbar\omega$$

$$\psi_0' = A e^{-\lambda |x|} = \sqrt{\lambda}\, e^{-\lambda |x|} = \sqrt[4]{\frac{m\omega^2}{2\hbar}} \exp\left[-\sqrt{\frac{m^2\omega^2}{2\hbar^2}} |x| \right]$$

$$V_{nm} = \int_{-\infty}^{+\infty} \psi^{+*} \hat{V} \psi_0^+ dx$$

$$\psi_0^+ = \sqrt[4]{i} \left(\frac{m^2\omega^2}{2\hbar^2} \right)^{\frac{1}{8}} \exp\left(-\sqrt{i} \left(\frac{m^2\omega^2}{2\hbar^2} \right)^{\frac{1}{4}} |x| \right)$$

$$\int_{-\infty}^{+\infty} \frac{m\omega^2 x^2}{2} \left(\frac{m^2\omega^2}{2\hbar^2} \right)^{\frac{1}{4}} \exp\left[-2\cos\frac{\pi}{4} \left(\frac{m^2\omega^2}{2\hbar^2} \right)^{\frac{1}{4}} |x| \right) =$$

$$= \frac{m\omega^2}{2} \left(\frac{m^2\omega^2}{2\hbar} \right) \cdot \left(2\cos\frac{3\pi}{4} \left(\frac{m^2\omega^2}{2\hbar^2} \right)^{\frac{3}{4}} \right)^{-1} =$$

$$= \frac{m\omega^2}{2\left(\frac{\sqrt{2}}{2}\right)^3\sqrt{\frac{m^3\omega^2}{2\hbar^2}}} = \frac{m\omega^2}{\frac{\sqrt{2}}{2}\frac{m\omega}{\sqrt{2}\hbar^2}} = 2\omega\hbar$$

$$V_{nn} = E_0' = 2\omega\hbar$$

1: $\psi_1 = B x e^{-\beta|x|}$

Нормуємо:

$$|B|^2 \int_{-\infty}^{\infty} x^2 e^{-2\beta|x|}\,dx = 1 \Rightarrow |B|^2\left(\int_{-\infty}^{0} x^2 e^{2\beta x}\,dx + \int_{0}^{\infty} x^2 e^{-2\beta x}\,dx\right) =$$

$$= |B|^2 \cdot \frac{1}{2\beta^3} = 1 \Rightarrow B = \sqrt{2\beta^3}$$

$$\hat{H}\psi_1 = -\frac{\beta\hbar^2}{2m}\frac{d}{dx}\left[\frac{d}{dx}\left(x e^{-\beta|x|}\right) + \frac{m\omega^2}{2} B x^3 e^{-\beta|x|}\right] =$$

$$= \frac{-\beta\hbar^2}{2m}\frac{d}{dx}\left[e^{-\beta|x|} - \beta e^{-\beta|x|}\frac{x}{|x|}\cdot x\right] + \frac{m\omega^2}{2} B x^3 e^{-\beta|x|} =$$

$$= \frac{-\beta\hbar^2}{2m}\left(-\beta e^{-\beta|x|}\frac{x}{|x|} - \beta e^{-\beta|x|}\frac{x}{|x|} - |x|\beta^2 e^{-\beta|x|}\frac{x}{|x|}\right) +$$

$$+ \frac{m\omega^2}{2} B x^3 e^{-\beta|x|} = \frac{\beta\hbar^2}{2m}\left(-2\beta e^{-\beta|x|}\frac{|x|}{x} - x\beta^2 e^{-\beta|x|}\right) +$$

$$+ \frac{m\omega^2}{2} B x^3 e^{-\beta|x|} = \frac{\beta\hbar^2}{2m}\beta e^{-\beta|x|}\frac{|x|}{x} + \frac{\beta\hbar^2}{2m} x\beta^2 e^{-\beta|x|} +$$

$$+ \frac{m\omega^2}{2} B x^3 e^{-\beta|x|}$$

$$J_1 = \int_{-\infty}^{+\infty} \left[\frac{\beta^2\hbar^2}{2m} \beta |x| e^{-2\beta|x|} + \frac{\beta^2\hbar^2}{2m} \beta^2 x^2 e^{-2\beta|x|} + \right.$$

$$\left. + \frac{m\omega^2}{2} \beta^2 x^4 e^{-2\beta|x|} \right] dx$$

Розглянемо окремо:

$$\int_{-\infty}^{+\infty} |x| e^{-2\beta|x|} dx = -\int_{-\infty}^{0} x e^{2\beta x} dx + \int_{0}^{\infty} x e^{-2\beta x} dx =$$

$$= \frac{x}{2\beta} e^{2\beta x}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} \frac{e^{2\beta x}}{2\beta} dx - \frac{x}{2\beta} e^{-2\beta x}\Big|_{0}^{\infty} + \int_{0}^{\infty} \frac{e^{-2\beta x}}{2\beta} dx =$$

$$= -\frac{1}{4\beta^2} e^{2\beta x}\Big|_{-\infty}^{0} - \frac{1}{4\beta^2} e^{-2\beta x}\Big|_{0}^{\infty} = -\frac{1}{4\beta^2} + \frac{1}{4\beta^2} = 0$$

$$\int_{-\infty}^{+\infty} x^2 e^{-2\beta|x|} dx = \frac{1}{2\beta^3}$$

$$\int_{-\infty}^{+\infty} x^4 e^{-2\beta|x|} dx = \int_{-\infty}^{0} x^4 e^{2\beta x} dx + \int_{0}^{\infty} x^4 e^{-2\beta x} dx =$$

$$= \frac{x^4}{2\beta} e^{2\beta x}\Big|_{-\infty}^{0} - \int_{-\infty}^{0} \frac{4x^3}{2\beta} e^{2\beta x} dx - \frac{x^4}{2\beta} e^{-2\beta x}\Big|_{0}^{\infty} + \int_{0}^{\infty} \frac{4x^3}{2\beta} e^{-2\beta x} dx$$

$$= -\frac{2}{\beta} \int_{-\infty}^{0} x^3 e^{2\beta x} dx + \frac{2}{\beta} \int_{0}^{\infty} x^3 e^{-2\beta x} dx = \frac{3}{\beta^2} \int_{-\infty}^{0} x^2 e^{2\beta x} dx +$$

$$+ \frac{3}{\beta^2} \int_{0}^{\infty} x^2 e^{-2\beta x} dx = \frac{3}{\beta^2} \cdot \frac{1}{2\beta^3} = \frac{3}{2\beta^5}$$

$$J_1 = \frac{\beta^2\hbar^2}{2m} \beta^2 \frac{1}{2\beta^3} + \frac{m^*\omega^2}{2} \beta^2 \cdot \frac{3}{2\beta^5} = \left| \beta = \sqrt{2\beta^3} \right| =$$

$$= \frac{\beta^2\hbar^2}{2m} + \frac{3m\omega^2}{2\beta^2}$$

Шукаємо min

$$\frac{dJ_x}{d\lambda} = \frac{\beta\hbar^2}{m} - 3\frac{m\omega^2}{\beta^3} = 0 = \frac{\beta^4\hbar^2}{m} - 3m\omega^2 = 0 \Rightarrow$$

$$\Rightarrow \beta = \left(\frac{3m^2\omega^2}{\hbar^2}\right)^{\frac{1}{4}}$$

$$E_1 = \frac{\sqrt{3}\, m\omega\hbar^2}{2m\hbar} + \frac{3m\omega^2\hbar}{2\sqrt{3}\, m\omega} = \sqrt{3}\,\omega\hbar$$

$$\psi_1 = Bx e^{-2\beta|x|} = \sqrt{2\beta^3}\, x \exp(-2\beta|x|) =$$

$$= \left(\frac{432\, m^6\omega^6}{\hbar^6}\right)^{\frac{1}{8}} \exp\left(-2\left(\frac{3m^2\omega^2}{\hbar^2}\right)^{\frac{1}{4}}|x|\right) =$$

$$= \left(\frac{12\sqrt{3}\, m^3\omega^3}{\hbar^3}\right)^{\frac{1}{4}} \exp\left(-\left(\frac{48m^2\omega^2}{\hbar^2}\right)^{\frac{1}{4}}|x|\right)$$

$$V_{nn} = \int_{-\infty}^{+\infty} \frac{m\omega^2}{2} x^3 2\beta^2 e^{-2\frac{4}{\sqrt{3}}\sqrt{\frac{m\omega}{\hbar}}|x|} dx =$$

$$= m\omega^2\beta^2 \int_{-\infty}^{+\infty} x^3 e^{-2\beta|x|} dx = m\omega^2\beta^2 \left[\int_{-\infty}^{0} x^3 e^{-2\beta|x|} dx + \right.$$

$$\left. + \int_0^{+\infty} x^3 e^{-2\beta|x|} dx\right] = m\omega^2\beta^2 \cdot \frac{3}{2\beta^4} \cdot \frac{\beta}{2} = \frac{3m\omega^2}{4\beta^2} =$$

$$= \frac{\sqrt{3}\, m\omega^2}{4\frac{m\omega}{\hbar}} = \frac{\sqrt{3}}{4}\omega\hbar$$

№2 $V(x) = -V_0\,\delta(x)$ В $t=0$ ямка змінилася вдвічі

При $t \le 0$: $V(x) = -V_0\,\delta(x)$

При $t > 0$: $V(x) = -2V_0\,\delta(x)$

ψ - ф-я для дельта ями: $V(x) = -\lambda\,\delta(x)$

$\psi(x) = \sqrt{\varkappa}\,e^{-\varkappa|x|}$, $\varkappa = \frac{m\lambda}{\hbar^2}$; $E_0 = -\frac{m\lambda^2}{2\hbar^2}$

Розглянемо наші випадки:

$t \le 0$: V_0

$\psi_1 = \sqrt{\varkappa}\,e^{-\varkappa|x|}\,e^{-\frac{iE_0 t}{\hbar}}$, $\varkappa = \frac{mV_0}{\hbar^2}$, $E_0 = -\frac{mV_0^2}{2\hbar^2}$

$t > 0$: $2V_0$

$\psi_2 = \sqrt{2\varkappa}\,e^{2\varkappa|x|}\,e^{\frac{-i4E_0 t}{\hbar}}$

Порахуємо імовірність знаходження в ямі

$$W = \left|\int_{-\infty}^{+\infty} \psi_1(V_0, t=0, x)\,\psi_2^*(2V_0, 0, x)\,dx\right|^2 =$$

$$= \left|\sqrt{2\varkappa}\int_{-\infty}^{+\infty} e^{-3\varkappa|x|}\,dx\right|^2 = \left|\sqrt{2\varkappa}\cdot\frac{2}{3\varkappa}\right|^2 \Rightarrow$$

$$\Rightarrow W = \left(\frac{2\sqrt{2}}{3}\right)^2 = \frac{8}{9}$$

№3 $Z: 1 \to 2$. Атом дейтерію, $t=0$ - розпад

$W_{1s \to 2p}$ - ?

а) $t \le 0$: 1s стан, $Z_1 = 1$:

$$\psi_{100}(\vec{r}, t \le 0) = \sqrt{\frac{Z_1^3}{\pi a^3}}\, e^{-\frac{Z_1 r}{a}}\, e^{-\frac{iE_1 t}{\hbar}}$$

б) $t \ge 0$: 2p стан, $Z_2 = 2$ (після розпаду)

$$\psi_{210}(\vec{r}, t \ge 0) = ir\sqrt{\frac{Z_2^5}{32\pi a^5}}\cos\theta\, e^{-\frac{Z_2 r}{2a}} \cdot e^{-\frac{iE_2 t}{\hbar}}$$

Обчислимо ймовірність переходу $1s \to 2p$:

$$W_{100 \to 210} = \left| \int_V \psi_{100}^*(\vec{r}, t=0)\, \psi_{210}(\vec{r}, 0)\, dV \right|^2 =$$

$$= \left| Z_1 = 1;\ Z_2 = 2;\ dV = r^2 \sin\theta\, d\theta\, d\varphi\, dr \right| =$$

$$= \left| \sqrt{\frac{1}{\pi a^3}}\, i \sqrt{\frac{1}{\pi a^5}}\, 2\pi \int_0^{+\infty}\int_0^{\pi} e^{-\frac{2r}{a}} r^3 \cos\theta \sin\theta\, d\theta\, dr \right|^2 =$$

$$= \left| \begin{array}{l} \text{Зробимо заміну} \\ \cos\theta = t \\ dt = \sin\theta\, d\theta \end{array} \right| = \left| \sqrt{\frac{1}{\pi^2 a^8}}\, i\, 2\pi \int_0^{+\infty}\underbrace{\int_{-1}^{1} r^3 e^{-\frac{2r}{a}} t\, dt}_{=0}\, dr \right|^2 = 0$$

$$W_{100 \to 210} = 0$$